The symposium will be organized by the Department of Logic and Philosophy of Science at the University of the Basque Country. The first circular will be sent out in May 1989.

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REPORTS

Symposium on the History of Modern Mathematics

Vassar College, Poughkeepsie, New York, June 20–24, 1988

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The Symposium on the History of Modern Mathematics, organized by David Rowe and John McCleary, was held at Vassar College on June 20–24, 1988. The meeting was sponsored by the National Science Foundation, Pace University, the History of Science Society, the International Commission on the History of Mathematics, and the International Union of the History and Philosophy of Science. Forty-five participants representing 12 countries took part in the symposium, which featured 29 lectures covering a variety of topics from the era of Euler and Clairaut to the onset of modern electronic computers.

The principal theme of the meeting, discussed in about half of the papers presented, concerned issues and ideas at the interface between pure and applied mathematics. This theme arose in several historical contexts ranging from early 19th-century France to Wilhelmian Germany and beyond to the United States during the Second World War. Other topics that received considerable attention included the emergence of projective geometry and the influence of projective concepts of geometry in general, Abel's theorem and its role in algebraic geometry, the geometric background of transformation group theory, contrasting notions of rigor in 18th- and 19th-century mathematics, the philosophical views of Cantor and Kronecker and subsequent developments in the foundations of mathematics, 19th-century American algebra, and number theory from Gauss to Hilbert. The following abstracts of these lectures are given in the order in which they were presented:

IVOR GRATTAN-GUINNESS (London): "Applied Mathematics versus Applied Mathematics among the Paris *Savants*, 1800–1840"

The French Revolution and Restoration periods saw a remarkable flowering of mathematics, and also the early stages of professionalization of science and the creation or reform of many state-supported institutions. The combination of these circumstances, together with the usual Parisian penchant for the *polemique* created an intense and competitive atmosphere in research (and also to some extent in teaching policy). Mathematics at this time was oriented around applications, but wide differences arose concerning the purpose and even the content of the mathematics to be pursued. This paper concentrated on some examples of disagreement between the *savants* about methods and modes of applying mathematics to the physical world. In particular, the striking differences in orientation between the general theoretical applier, whose prototype was Lagrange, and the engineer *savant*, typified by Lazare Carnot, were shown.

JESPER LÜTZEN (Copenhagen): "Liouville's Differential-Geometric Interpretation of the Principle of Least Action"

It is fairly well known that it was through Joseph Liouville that Gauss's revolutionary ideas on differential geometry became familiar in France. What has not been generally appreciated, however, is that no one before Riemann had a better understanding of the fundamental significance of Gauss's conceptions than did Liouville. This paper discussed his use of differential geometry in a short article on the principle of least action in which Liouville derived the Hamilton–Jacobi equations of classical mechanics by treating the motion of a particle in a force field as equivalent to regarding the same particle in a force-free setting where it moves along a geodesic curve of a certain surface. Liouville also went beyond Minding and Bonnet by giving an intrinsic definition of the geodesic curvature of a curve on a surface. His sophisticated use of intrinsic ideas is also documented in the manuscript notes of his 1851 lectures held at the Collège de France.

THOMAS ARCHIBALD (Wolfville, Nova Scotia): "Physics as a Constraint on Mathematical Research: The Case of Potential Theory and Electrodynamics"

The profound effects that developments in physics exercised on the direction of mathematical research are particularly well illustrated by the relationship between electromagnetic theory and mathematical potential theory in Germany during the period 1840 through 1880. Electrodynamical research during this period was based on a model proposed by Wilhelm Weber, which used an action-at-a-distance force depending on the velocities of the interacting particles. This dictated that the potentials for these forces should be velocity-dependent. The initial attack on this problem was a mathematical one, namely, to generalize the notions of potential and potential energy suitably while retaining an energy conservation law. Riemann was the first to attempt to free Weber's force from velocity dependence in his *Beitrag zur Elektrodynamik* of 1858. Its essential idea, that the potentials should be treated as propagated with finite velocity, was later pursued by Carl Neumann using variational methods and Lagrangian dynamics. The increasing acceptance of Maxwell's theory rendered these and similar efforts by E. Schering, R. Clausius, and H. Helmholtz obsolete from the standpoint of physical theory. By the mid-1870s research into potential theory turned increasingly to pure mathematical questions in two closely related areas: the theory of boundary-value problems and complex analysis.

EBERHARD KNOBLOCH (West Berlin): "Mathematics at the Berlin Technische Hochschule"

The Berlin Technische Hochschule was established in 1879 as a result of a merger between the Berlin Bauakademie, founded in 1799, and the Gewerbeakademie, which was founded in 1821. Mathematics was taught as an auxiliary subject at these institutions. The teachers at the Bauakademie, which from the beginning had a higher academic status, were often professors or lecturers from the University of Berlin: J. P. Gruson, P. G. Lejeune-Dirichlet, M. Ohm, F. Minding et al. K. Weierstrass, B. Christoffel, and S. Aronhold were among the mathematicians who taught at the Gewerbeakademie. In 1916 the Bergakademie, which was founded in 1770, was incorporated into the Berlin Technische Hochschule, giving the latter three new mathematicians—F. Kötter, A. Kneser, and E. Jahnke. Three unique characteristics distinguished the growing field of mathematics at the Technische Hochschule:

1. a close connection with mechanics and other applied areas;
2. research in geometry, analysis, and algebra;
3. an important role concerning the foundation and development of the Berlin Mathematics Society.

ERHARD SCHOLZ (Wuppertal): "Crystallographic Symmetry Concepts and Group Theory"

Preceding the first explicit use of the group concept in geometry by Camille Jordan in 1869 were the investigations of A. Bravais on the structure of crystals and regular point systems in space, which were undertaken between 1849 and 1851. In fact, Bravais not only classified finite symmetry systems (and thus implicitly the finite orthogonal groups that act on 3-space), but he also began to consider the much more complex symmetry systems of point or molecular patterns in 3-space (implicitly the symmorphic space groups of crystallography). Bravais, on the other hand, drew heavily on Haüy's atomistic theory of crystal structure, which was dominant in France since the early 19th century, as well as certain aspects of the dynamistic school of crystallography founded by C. S. Weiss in Germany. Beginning with this as background, this paper traced the role of concepts that arose in studies of crystallographic symmetry in the early phases of geometric group theory.

BRUCE CHANDLER (New York): "Knot Theory: The Scientist's versus the Mathematician's View"

Knowledge of knot structures can be found in a variety of sources from at least the time of the Renaissance. They first became important in physics when William Thomson proposed the vortex theory of the atom. Through Thomson's colleague, P. G. Tait, the investigation of knot types took on a life of its own long after Thomson's model had ceased to elicit much interest on the part of physicists. Tait's purely combinatorial techniques, however, proved to be an insufficient tool for recognizing when two knots are identical. It was not until after the development of combinatorial group theory in the mid-1880s and Poincaré's subsequent creation of the fundamental group, an important topological invariant, that knot theory could emerge in the sense that we know it today. These developments began with the work of Max Dehn and were greatly enhanced by J. W. Alexander, whose name is connected with a certain polynomial expression that is an invariant for all knots. In recent years these purely mathematical developments have again found applications in cell biology and a variety of other scientific fields.

GERT SCHUBRING (Bielefeld): "Pure and Applied Mathematics in Divergent Institutional Settings within the German States and the Role of Felix Klein in the Evolution of a New Balance"

The institutional historiography of 19th-century German mathematics tends to concentrate on developments in Prussia. This may be ascribed partly to Prussian mathematicians' use of a new institutional system based on reforms in university and secondary school education and the creation of a "disciplinary–professional complex" that enabled Prussia to overtake France as the leading nation in mathematics in the 1830s. On the other hand, this Prussian model tended to enhance an ever-growing tendency toward specialization in mathematical research and a concomitant disdain for applications. However, in other German states, where no comparable research imperative had become institutionalized, mathematics developed rather differently in both universities and secondary schools. Moreover, other types of institutions were able to flourish in these settings, particularly polytechnical schools and colleges, which promoted otherwise neglected fields like synthetic and descriptive geometry, and applications in general. This paper considered some of the tensions underlying this situation and the efforts of Felix Klein to reorient pure mathematics and reconcile these divergent forces at the end of the century.

Renate Tobies (Leipzig): "On the Contribution of Mathematical Societies to Promoting Applications of Mathematics in Germany"

The statement once made by E. E. Kummer that "applied mathematics is dirty mathematics" reflects a viewpoint that was prominent among German mathematicians in the last decades of the 19th century. Before mathematics could attract more attention in Germany and be utilized in other fields, it was necessary to overcome this purist attitude. Those who were interested in bringing about these changes, which were characteristic of mathematical developments around the turn of the century, sought to do so through the channels of scientific societies and other organizations. Foremost among these was the Deutsche Mathematiker-Vereinigung, founded in 1890 within the context of the Gesellschaft deutscher Naturforscher und Äerzte, but representatives from industry were also interested in promoting these developments through organizations like the Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik. Newly uncovered sources reveal that their first priority was to bring about improvements in teacher training that would be oriented toward applications of mathematics; to this end a number of commissions were established. In Germany and elsewhere, Felix Klein was a driving force behind many of these developments and the founding of organizations to promote education in applied mathematics.

Amy Dahan-Dalmédico (Paris): "Works of Poisson and Cauchy on the Theory of Waves and Their Mathematical Development, 1815–1825"

The work of Cauchy and Poisson on wave motions in a canal was occasioned by a prize competition set forth by the Académie des Sciences in 1815. Both mathematicians adopted a phenomenological approach to this problem, and there was no inherent conflict between their respective physical assumptions. They also utilized similar mathematical techniques: Fourier transforms and series expansions. Nevertheless, they initially arrived at qualitatively different results. In particular, Cauchy's analysis led to uniformly accelerated waves, whereas Poisson's included these as well as waves in uniform motion. These conflicting results made Cauchy aware of certain pitfalls that must be avoided when manipulating Fourier series. Poisson's memoir of 1919 also presented a complete integration of the equations for the wave surfaces, which decisively influenced Cauchy's theory of partial differential equations.

Roger Cooke (Burlington, VT): "An Historical Interpretation of Abel's Theorem"

The famous theorem of Abel, as later work was to show, had enormous ramifications. The form in which Abel originally stated the theorem left many aspects of its meaning unclear. In particular it is not at all evident how Abel himself came to formulate his theorem. In their classic study of 1894, "Die Entwicklung der Theorie der algebraischen Funktionen in älterer und neuerer Zeit," Alexander Brill and Max Noether offered what they regarded as a plausible interpretation as to how Abel conceived of this grandiose result, arguing that it was derived from the case for elliptic integrals. After presenting a criticism of their analysis, this paper set forth a new interpretation that is both simpler and more easily motivated.

UMBERTO BOTTAZZINI (Bologna): "Cauchy and the Emergence of Complex Function Theory in the Nineteenth Century"

Cauchy's work in complex function theory may naturally be divided into two periods. The first, which begins with his 1814 paper on the order of integration in double integrals, was dominated by his use of complex variables to determine real integrals. Beginning in 1831, Cauchy's interests shifted to the investigation of complex functions by means of power series. The focus of his early work was his "calcul des residues," which he set forth in a paper of 1822. The calculus of residues was far more central to his conception of the theory than was the Cauchy integral theorem. Cauchy was well aware of contemporary discussions on the geometry of complex numbers, but he took little interest in these matters, since for him imaginary quantities were simply a tool for dealing with functions of a real variable.

KARINE CHEMLA (Paris): "The Treatment of Duality in Spherical Trigonometry"

Spherical trigonometry was a subject that interested most of the important mathematicians of the 18th and early 19th centuries, including Euler, Lagrange, Lacroix, Laplace, Hachette, and Monge. Already during the 18th century, developments in this field, namely, analytical theories, had focused on the phenomenon of duality. Duality arguments in spherical trigonometry can already be found in Euler, and Lagrange often derived dual theorems although he almost never constructed dual algebraic proofs, as Euler did systematically. Gergonne, who emphasized the importance of duality in projective geometry, was partly motivated by his knowledge of its efficacy within the field of spherical trigonometry. This paper examined these earlier investigations to analyze and compare the ways in which duality was treated by various authors.

JOAN RICHARDS (Providence): "Legendre and his *Eléments de Géométrie*"

This paper began by pointing out that the concept of "rigor," which has been a byword for good mathematics since the time of Gauss and Cauchy, has not always been seen as central to sound mathematics. In addition, there have been times when it has had implications within mathematics as negative as those that it still has in common English and French usage. In France, the central importance of rigor to valid mathematical work was accepted only slowly and against considerable opposition in the first decades of the 19th century. In the 18th century, different aspects of the subject were often seen as more important. This can be seen clearly in basic geometry texts, where rigor could always have been attained, but it was not generally emphasized. In his 1841 *Elémens,* Clairaut explicitly abandons the rigorous artifices of Euclid in favor of an approach rooted in geodesy. He characterizes his presentation as a "natural" one, reflecting the natural psychological and historical progression of geometrical ideas. Clairaut's notion of the natural can be more fully understood by looking at D'Alembert's treatment of geometry and mathematics in the *Encyclopédie,* where he explores the balance between the natural and the rigorous approaches. D'Alembert recognizes both values, but comes down clearly on the side of the natural. This faith in the veracity of the understanding that flows from historico-psychological development is strikingly absent in Legendre's *Géométrie* of 1794, in which rigor is central and the natural banished. This change reflects a more pessimistic perception of the products of both history and human nature. Just as Clairaut's and D'Alembert's views seem to have mirrored their optimistic times, Legendre's appear to reflect the difficulties of his. In the subsequent turbulent decades these different views were debated in French mathmatics; a great many changes in attitude had to be negotiated before rigor was accepted as the ultimate value in mathematics.

JEREMY GRAY (London): "On Projective and Algebraic Geometry, 1850–1890"

The historical literature on algebraic geometry offers a rich plethora of techniques and results, but it fails to offer many clues to what motivated the intense activity that occurred in this field throughout the last half of the 19th century. This paper traced some of this neglected background, emphasizing the larger ideas that guided work in this field. Among such were (1) the need to incorporate Abel's theorem and its consequences, (2) the search for a suitable theory of transformations—the birational transformations—that could remove unwanted singularities, hence (3) the desire to develop a theory of birational invariants to go along with such projective invariants as Plücker's formulas, and (4) the generalization of concepts from curve theory, like genus, to algebraic surfaces. These themes and others came strongly into play through the work of Brill, Noether, and especially the Italian geometrical school.

GREGORY NOWAK (Princeton): "Riemann's *Habilitationsvortrag* and the Synthetic *a priori* Status of Geometry"

Bernhard Riemann's *Habilitationsvortrag* of 1854 (first published in 1868) has long been regarded as a foundational paper for topology and differential geometry. Over the years it has been seen as an attempt by Riemann to prophesy the future direction of physics or to determine a new axiomatic foundation for geometry. This paper argues that Riemann was strongly concerned with the philosophical foundations of geometry and that he sought to reevaluate them in light of his work in differential geometry. In particular, Riemann was aware of the implications of his work for the Kantian conception of space and the existence of synthetic *a priori* propositions, and he organized his exposition in the form of a refutation of Kant's attempt to grant Euclidean geometry the status of *a priori* truth. The connection between Riemann's *Habilitationsvortrag* and the Kantian conception of space has been observed previously; this paper seeks to make the connection as an explicit part of Riemann's motivation in writing this work.

DIRK STRUIK (Belmont, MA): "Schouten, Levi-Civita, and the Emergence of Modern Tensor Analysis"

The roots of tensor analysis lie in Riemann's theory of quadratic differential forms as elaborated by Christoffel and Lipschitz. In 1884 Ricci-Curbastro presented his absolute differential calculus by combining this work with Beltrami's theory of differential parameters. Ricci's pupil, Tullio Levi-Civita, took another decisive step forward in developing what is now known as the theory of tensors by introducing the notion of parallelism. Elie Cartan and J. A. Schouten, who discovered parallelism independently of Levi-Civita, also contributed much to the conceptual development of tensor theory. The key event that brought the field into prominence, however, was of course Einstein's creation of the general theory of relativity and the subsequent discovery of the gravitational field equations in 1915. The speaker, a participant in these developments, recounted this background and the years he worked together with Schouten and Levi-Civita, characterizing their personalities and chief mathematical accomplishments.

DAVID ROWE (Pleasantville, NY): "Klein, Lie, and the Emergence of the Erlangen Program"

This paper examined the early work of Felix Klein and Sophus Lie leading up to the publication of the Erlangen Program in 1872. The initial researches of Klein and Lie were closely related to Plücker's work in line geometry, but they quickly broadened their efforts to cover other related fields. Lie discovered a number of new ideas that generalized Plücker's original concepts, and he eventually applied these to create a new theory of differential equations that owed much to the ideas of Monge. Klein and Lie shared a dynamic approach to geometry which they studied in conjunction with a suitably chosen family of transformations that acted on a space or certain subspaces. They also shared a certain predilection for seeking out analogies between seemingly unrelated mathematical structures (e.g., line geometry and complex projective spaces, or line geometry and sphere geometry). The most important single breakthrough that occurred during their early collaboration was Lie's discovery of the line-to-sphere transformation, which he showed has the key property that it maps the asymptotic curves of one surface onto the lines of curvature of another. Both Lie and Klein regarded this result, which they returned to often in several papers written between 1870 and 1872, as a bridge between German line geometry, a field dominated by projective concepts, and the metric geometry developed by the French school.

THOMAS HAWKINS (Boston): "Lie, Klein, and the Emergence of the Theory of Transformation Groups"

According to Lie, his theory of groups was born in October 1873. To appreciate why the discoveries of that fateful October proved so consequential as to prompt Lie to devote himself to the creation of the theory, they must be seen as the culmination of certain tendencies within his prodigious geometrically oriented research activities during the preceding 4 years, when he worked in close contact with Felix Klein. During that period Lie's research progressed through three phases, each of which contributed in a special way to the birth of the theory of continuous transformation groups: (1) investigations related to the tetrahedral complex (1869–1870); (2) the discovery and exploitation of the "line-to-sphere map" (1870–1871); (3) work on a geometrical theory of differential equations in the spirit of Monge (1871–1873). The nature and significance of these three phases were briefly discussed. Within the context of that discussion, the important role played by Klein in directing Lie's attention to group-theoretic aspects and implications of his work during the first two phases was emphasized. The possibility that Jordan's memoir of 1869 on groups of motions, cited by both Lie and Klein in papers of 1871, exerted a significant influence upon the events leading to October 1873 was deemed unlikely in view of that context.

LARRY OWENS (Amherst): "Mathematicians at War: The Applied Mathematics Panel of the Office of Scientific Research and Development"

The outbreak of war in 1939 caused American mathematicians, like other scientists, to explore the ways in which they might contribute to the national defense. The pressure of war, however, did more than set American mathematicians against the enemy; it set Americans against each other in arguments that concerned the responsible exercise of power in the mathematical community, the character of applied mathematics and the training best suited to it, and the role mathematicians would continue to play in the postwar period. This paper centered on the Applied Mathematics Panel of the Office of Scientific Research and Development headed by Warren Weaver, a strong advocate of *useful* applied mathematics. A wide variety of conflicting views and attitudes were expressed by the panel members, who included Richard Courant, Marston Morse, Oswald Veblen, and T. C. Fry.

WILLIAM ASPRAY (Minneapolis): "Early Computer-Oriented Numerical Analysis, Monte Carlo Techniques, and Their Applications to Hydrodynamics"

The development of the high-speed electronic computer offered major new opportunities to the applied mathematician in the 1950s. With the automation of the computational process, many problems of the physical world could be solved by approximation methods with fewer unrealistic simplifications. These opportunities revived the study of numerical analysis. The effects of automatic computation on round-off errors, fast methods for matrix inversion, and approximation techniques for partial differential equations were investigated. Entirely new statistical methods, like Monte Carlo, were developed. These efforts opened new areas to mathematical investigation, in particular hydrodynamical problems common to weapons design, aeronautical engineering, and geophysical science. This paper discussed these developments, with special attention to activities at major scientific computation centers like Los Alamos laboratory, the Institute for Numerical Analysis, and the Institute for Advanced Study.

CHRISTIAN HOUZEL (Paris): "Elliptic Functions and the Solution of the Quintic Equation"

That the general equation of the fifth degree is not algebraically solvable was first discovered in 1799 by Ruffini, who gave a proof that was essentially correct, but incomplete. He implicitly assumed that all the auxiliary quantities that appear in the eventual resolution formulas are rationally expressible as functions of the roots of the equation, a property first proved by Abel in 1823. For the general fifth-degree equation, the theory of elliptic functions provided the means for constructing the analog to angle trisection for the general cubic. The first to employ this approach was Betti in 1854. The work of subsequent researchers—Hermite, Kronecker, Brioschi, and Klein—was related to the earlier work of Jacobi on modular functions. Galois, in his last letter to Chevalier, had already indicated the structure of the corresponding "Galois group" for the modular equations, results that were later proven by Betti.

HELENA PYCIOR (Milwaukee): "Early American Algebra through Benjamin Peirce"

In colonial America students learned mathematics from imported British textbooks. Following the War of Independence, however, Americans became more familiar with French ideas and institutions. The French influence helped shape the Military Academy at West Point; according to standard histories, it also led to the adoption of French mathematical textbooks for the American classroom. This paper used four influential American algebra textbooks as bases for a reexamination of the French influence on mathematics in the early American Republic. It showed that by the 1810s Americans were weighing the relative merits of two distinct algebraic styles, the British synthetic style and the French analytic style. The Americans, however, did not universally opt for the French analytic style and French textbooks. Rather, one of the most popular American algebras of the period, Jeremiah Day's *Introduction to Algebra,* followed the British synthetic style. Even Charles Davies's *Elements of Algebra: Translated from the French of M. Bourdon*—the most successful of the so-called French "translations"—was not a faithful rendering of the original textbook. Davies took great liberties with the original, as he sought to unite the French "scientific discussions" with the British "practical methods." Benjamin Peirce's *Elementary Treatise on Algebra* essentially ignored both styles. In fact, of the four textbooks, the only one that adhered rigidly to the French analytic style was John Farrar's translation of Lacroix's *Elémens d'algèbre*. In general, the paper established that American educators of the early Republic pursued the active roles of clarifying, judging, adapting, and synthesizing British and French mathematics.

KAREN V. H. PARSHALL (Charlottesville): "J. J. Sylvester and the Johns Hopkins Mathematical School"

In virtually every discussion of the critical period of change in the history of American science, that is, the period from 1875 to 1900, the Johns Hopkins University figures prominently as the first university founded to provide advanced as well as undergraduate training in the sciences and humanities. The first professor of mathematics at Hopkins, James Joseph Sylvester, consciously sought to build a mathematical school at his institution that would produce theorems rivaling those of the well-established schools of Europe. Sylvester strove to meet the institutional goals of high-level instruction and research productivity by establishing a kind of mathematics laboratory with himself as director. From 1881 to 1883 he and his students established a so-called "constructive" theory of partitions. The main fruits of their labor appeared in Sylvester's paper, "A Constructive Theory of Partitions, Arranged in

Three Acts, an Interact, and an Exodion." The novelty of the results and the interest that this work generated, principally in England and France, provide concrete evidence of the crucial role played by the Johns Hopkins University in the early years of America's coming of age in mathematics.

WALTER PURKERT (Leipzig): "Cantor's Views on the Foundations of Mathematics"

In 1870 when Cantor proved his uniqueness theorem for Fourier series representations he was already involved in debates about the foundations of mathematics between the Weierstrassian school on the one hand and Kronecker on the other. In 1872 he was able to extend this theorem by introducing the concept of the derivative of a point set, an idea that led him to the notion of transfinite ordinals in the form of derivative orders of point sets. After sketching the further development of Cantor's mathematical ideas, this paper focused on his philosophical views. Cantor's letters to Hilbert, preserved in the Hilbert *Nachlass* in Göttingen, reveal that in 1883, i.e., 20 years prior to Russell's publications, Cantor was already well aware that both the totality of all ordinals and the totality of all cardinals form inconsistent sets. The reason he was not disquieted by this discovery lies in his Platonistic outlook on mathematics in general together with his identification of inconsistent systems with what thinkers like Leibniz and Spinoza called the "Absolute." Cantor himself called such a system the "Transfinitum in Deo," which he strictly distinguished from transfinite sets, the only infinite sets that the human intellect is capable of conceiving.

HAROLD M. EDWARDS (New York): "Kronecker's Views on the Foundations of Mathematics"

In our day, the phrase "foundations of mathematics" has become synonymous with "set theory." The key to understanding Kronecker's views on the foundations of mathematics is the realization that these views have nothing to do with set theory. They deal with what we call today algorithms. Kronecker objected to the use of "arbitrary infinite sequences" in mathematics, and consequently objected to the notion of "arbitrary real numbers." In his view, the infinite sequences considered in mathematics should be specified algorithmically so fully that they do not in fact leave the domain of the finite. In rejecting the actual infinite, Kronecker was following the tradition of his predecessors from Aristotle and Archimedes to Gauss. Although it has often been said that Kronecker succeeded in making a great contribution to mathematics only because he abandoned his principles, this opinion has never been supported by any examples. The fact is that his mathematical works do always deal with specific mathematical constructs using specific algorithmic techniques that conform to the requirements he advocated.

GREGORY H. MOORE (Hamilton): "Towards a History of the Continuum Problem"

In 1878 Cantor asserted that there are only two possible classes of linear point sets: countable sets and sets that are equivalent to a whole interval. Five years later he formulated a stronger form of the continuum hypothesis (CH), asserting that the continuum has the same power as that of the second number class. This paper was concerned with the historical reception of this problem. In the course of this survey, six different approaches to the continuum hypothesis were delineated: (1) prove CH for a restricted class of sets (Cantor–Bendixson, Hausdorff, Alexandrov); (2) generalize CH (Hausdorff et al.); (3) derive consequences (Luzin, Polish school); (4) propose alternatives (Luzin et al.); (5) seek equivalent statements (Sierpinski et al.); (6) derive CH from other axioms. The paper also stressed the importance of Hilbert's Paris address, in which the continuum hypothesis appears as the first of Hilbert's 23 problems. It closed by summarizing the work of later logicians with regard to the consistency, relative consistency, and independence of the continuum hypothesis within the Zermelo–Fraenkel axiom system for set theory.

JOHN MCCLEARY (Poughkeepsie): "A Theory of Reception for the History of Mathematics"

This paper outlined an adaptation of the methodological viewpoint of reception aesthetics, borrowed from literary history, to the history of mathematics. Treating a work of mathematics as a piece of literature that appears in response to the times in which it is written, by the author educated and living in this time for an audience of the time, adds to the historical picture in a way that enriches the historicist view and avoids its pitfalls. The paper began by sketching and criticizing other accepted viewpoints toward the history of mathematics. Next the theory of reception is adapted from the work of Hans Robert Jauß. The contrasts between mathematics and literature are then made explicit and from this it emerges that reception reflects some of the concerns of the audience of the history of mathematics; that is, reception focuses attention on the kinds of problems that interest the historian of mathematics. Finally, examples and possible projects were discussed for which this tack on historiography appears to be well suited.

OLAF NEUMANN (Jena): "On the History of Binary Quadratic Forms from Euler to Gauss"

The central problem in the theory of binary quadratic forms is to decide under what conditions the diophantine equation $ax^2 + bxy + cy^2 = n$ has solutions and to determine what these solutions are. This problem is closely related to the law of quadratic reciprocity, which was discovered by Euler but first proven by Gauss. Lagrange showed that in certain cases it was possible to compose the equivalence classes of binary quadratic forms in a commutative fashion, and in his *Disquisitiones Arithmeticae* Gauss followed up on this by exhibiting a general law of composition, showing, in effect, that the equivalence classes form an abelian group. Dedekind later gave a geometric interpretation for these classes and their composition by using point lattices in the plane.

GÜNTHER FREI (Zurich): "Heinrich Weber and the Emergence of Class Field Theory"

Following up research by James Bernoulli on infinite sums of reciprocal powers $\sum_{n=1}^{\infty} 1/n^k$, $k \in \mathbb{N}$, Euler introduced around 1737 the zeta function $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$, $s \in \mathbb{R}$, $s > 1$. Euler established or discovered many of the fundamental properties of the zeta function and realized its fertility in applications to number theory: the product formula (1737), divergence of the series $\sum_p 1/p$ over all primes p (1737), the relation of $\zeta(2k)$, $k \in \mathbb{N}$, to the Bernoulli numbers (1739), and the functional equation (1749, proved by Riemann in 1859). To extend Euler's result on the divergence of $\sum_p 1/p$ to primes of the form $p \equiv a$ modulo m for fixed $a, m \in \mathbb{N}$ with $(a, m) = 1$, Dirichlet introduced in 1837 congruence characters modulo m

$$\chi: \mathbb{Z} \to \mathbb{C} \qquad \text{and} \qquad L\text{-series}$$

$$L(s, \chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}, \qquad s \in \mathbb{R}, s > 1$$

attached to them. The main step in the proof of the divergence theorem then consisted in showing that $L(1, \chi) \neq 0$ for a real character $\chi \neq \chi_0$ (χ_0 the principal character). The extension of the zeta function to number fields K is due to Dedekind (1871), who established the product formula and determined the residue at $s = 1$ for such zeta functions ζ_K, which yields a general class number formula and the density of prime ideals in ideal classes. Weber combined Dedekind's zeta function with Dirichlet's L-series to define L-series $L(K, \chi, s)$ attached to characters of congruence class groups C_M modulo M, where M is an ideal in the field K. He was then able to show that there are infinitely many prime ideals of first degree in each class of C_M under the hypothesis that the ideals are equidistributed among the classes of C_M and that there exists a so-called class field A of K to C_M. This notion was introduced by Weber, and the decisive step in his proof consisted in showing that $L(K, \chi, 1) \neq 0$ for $\chi \neq \chi_0$, χ a character of C_M, if there exists a class field A over K to C_M.